Coherent synchrotron radiation transient effects in the energy-dependent region

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Coherent synchrotron radiation (CSR) is a well known phenomenon that originates from coherent superposition of electromagnetic waves by ultrarelativistic electrons. CSR longitudinal effects during the passage of a Gaussian beam from a straight to a circular path have often been studied in a regime in which they are energy independent. Nevertheless, the approximations used in such a regime may fail in several practical situations, as in the case of low-energy injectors or for small-wavelength structures within the bunch distribution in CSR-related instability problems. These situations demand a deeper investigation of longitudinal transient effects in the region where the approximations above are no longer valid: a strong γ dependence is found, and described in this paper, in the rate of energy change induced by CSR during the transient of a Gaussian bunch between a straight and a circular path, which was studied with the help of the authors' previous work. Results show that the overall CSR longitudinal effects, in this case, are reduced. One of the outcomes of previous work by Saldin *et al.* was extended to this situation and very good agreement between the two studies was found.

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I. INTRODUCTION

In the last few years, electron acceleration technology has focused more and more on the production of very short, high-charge bunches of electrons, which are expected to be used in x-ray self-amplified spontaneous emission free-electron lasers. Similar beams are also being considered for production of femtosecond radiation pulses by simpler schemes based on Cherenkov and transition radiation [1]. However, their production and utilization may prove difficult due to collective radiative effects.

Consider a relativistic electron bunch moving along a circular trajectory. In the part of the spectrum where the photon wavelength becomes comparable with the size of the bunch, electromagnetic waves emitted by individual particles have small phase differences. As a result, they add up coherently, thus leading to a quadratic dependence of the intensity of radiation on the number of electrons in the bunch [2]. This number is typically 10^8–10^{10}, which explains the high magnitude of the effect. This will be referred to as the steady state coherent synchrotron radiation, or CSR (see [3] for a detailed discussion). Due to the geometry of the system, and in the approximation of a rigid beam, the distance between retarded radiators and the present test particles (lying within the moving bunch) is stationary.

In contrast to the above case of steady state CSR, in any beam optics system, like bending magnets, magnetic chicanes, or others, transient collective phenomena also take place, where the signal retardation no longer obeys a stationary relation. The problem of a transient between a straight path and a region of constant bending radius has been a matter of active theoretical research (see, for example, [4] and [5]) in the last few years, this subject being of fundamental importance for a proper understanding of CSR phenomena. Other more general works, both numerical and theoretical followed (see, among others, [6–17]).

In this paper a very important result obtained in [4] will be used, namely, an expression for the rate of energy change of an electron as a function of its position in a rigid bunch with a generic distribution function (and, in particular, a Gaussian distribution) at a fixed position (that is, at a fixed time) from the entrance of a hard edge magnet. This result generalizes the outcome of [3], which deals with the steady state only, by covering the transient case too. Some of the articles cited above were based, in particular, on this result in the steady state [12–15] or on both steady state and transient cases [16,17].

All these investigations deal with ultrarelativistic beams in which the value of the Lorentz factor γ is always above a certain threshold, as the result in [4] upon which they are based is derived in that approximation. Nevertheless, in several practical situations, like, for example, the case of low-energy injectors (see, for example, [18]), we deal with ultrarelativistic beams for which the value of γ, in combination with the bunch length and the choice of the trajectory, may be low enough so that the latter approximation fails. On the other hand, recent research (see [13–15]) has shown that there is reason for concern about beam instabilities induced by CSR in storage rings as well as in bunch compressors. In fact, CSR has been shown to amplify small sinusoidal perturbations of the bunch distribution during the evolution of the beam: once again, one should make sure that the perturbation wavelength (in combination with the beam energy and the geometry of the magnetic system) satisfies the approximations above before using the results in [4] (or, alternatively, for the steady state regime, in [3]).

These observations demand a deeper investigation of CSR effects in such situations. In the present paper, and with the help of [8], the problem of a Gaussian bunch crossing a transient between a straight path and a region of constant bending radius in the low-energy region has been studied. The results show a strong dependence of the CSR longitudinal force on the Lorentz factor.

Further on, the formula in [4] for the rate of energy change of an electron is extended in such a way that it is

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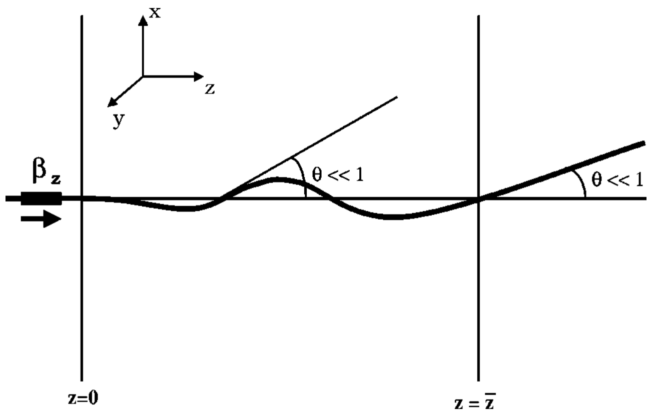

FIG. 1. Schematic of a particle trajectory in the small-angle approximation.

valid independently of the choice of γ, and it is used to treat the same problem: a comparison between the two studies shows very good agreement.

This work is organized as follows. In Sec. II a brief review is given of the main results of the small-angle approximation method proposed in [8], and it is applied to a low-γ transient case. Then, in Sec. III, an extension of the formula in [4] to the γ-dependent region is presented, and perfect agreement is shown between such an extension and the previous results in this article. Finally, Sec. IV is dedicated to conclusions.

II. SMALL-ANGLE APPROXIMATION AND ITS APPLICATION TO A TRANSIENT IN THE LOW-γ REGION

An expression for the rate of energy change of a test particle within a rigid, one-dimensional (1D) bunch with generic density distribution function moving along a generic trajectory was recently presented in [8]. The expression was derived by means of a consistent use of the paraxial approximation explained in Fig. 1. As explained in [8], this approximation is applicable to a very wide class of trajectories.

Referring to [8], and therefore to the geometry in Fig. 1, z_0 is defined as the present z position of a test electron and z as the present z position of a source electron; let $\Delta z = z_0 - z$ and always consider $\Delta z > 0$, the contributions to CSR effects from the case $\Delta z < 0$ being negligible (see [4,8]). Then, always following [8], the rate of energy change for a test particle can be written as

$$\frac{d\mathcal{E}}{d(ct)} \simeq \frac{e^2}{4\pi\epsilon_0 c^2}\int_{-\infty}^{z_0} dz'\{[C]+[R]\}\lambda(\Delta z), \qquad (1)$$

where λ is the bunch density. Here $[C]$ and $[R]$ stand for the terms due to the Coulomb and radiative parts of the retarded fields, namely,

$$[C] \equiv \frac{2c}{(z_0-z')^2} \times \left\{ \frac{1-\gamma^2[\boldsymbol{\beta}_\perp(z_0)-\boldsymbol{n}_\perp]^2+\gamma^2[\boldsymbol{\beta}_\perp(z_0)-\boldsymbol{\beta}_\perp(z')]^2}{\{1+\gamma^2[\boldsymbol{n}_\perp-\boldsymbol{\beta}_\perp(z')]^2\}^2} - \frac{1+\gamma^2[\boldsymbol{n}_\perp-\boldsymbol{\beta}_\perp(z')]^2}{\left[1-\gamma^2\boldsymbol{n}_\perp^2+\gamma^2(z_0-z')^{-1}\int_{z'}^{z_0}\boldsymbol{\beta}_\perp^2(\zeta)d\zeta\right]^2} \right\}, \qquad (2)$$

$$[R] \equiv 2\gamma^2 \frac{\dot{\boldsymbol{\beta}}_\perp}{(z_0-z')\{1+\gamma^2[\boldsymbol{n}_\perp-\boldsymbol{\beta}_\perp(z')]^2\}^2} \times ([\boldsymbol{n}_\perp-\boldsymbol{\beta}_\perp(z')]\{1+\gamma^2[\boldsymbol{\beta}_\perp(z_0)-\boldsymbol{\beta}_\perp(z')]^2 - \gamma^2[\boldsymbol{n}_\perp-\boldsymbol{\beta}_\perp(z_0)]^2\} - [\boldsymbol{\beta}_\perp(z_0)-\boldsymbol{\beta}_\perp(z')] \times \{1+\gamma^2[\boldsymbol{n}_\perp-\boldsymbol{\beta}_\perp(z')]^2\}), \qquad (3)$$

where $\boldsymbol{\beta}_\perp$ and $\dot{\boldsymbol{\beta}}_\perp$ are, respectively, the dimensionless velocity and its time derivative at the retarded time t' in the direction orthogonal to z. Finally, $\boldsymbol{n}_\perp$ is given by

$$\boldsymbol{n}_\perp = \frac{1}{(z_0-z')}\int_{z'}^{z_0} d\zeta \boldsymbol{\beta}_\perp(\zeta). \qquad (4)$$

The information about the retardation condition is automatically included in the expression for $\lambda(\Delta z)$, being

$$2\Delta z \simeq \frac{(z_0-z')}{\gamma^2} + \int_{z'}^{z_0} d\zeta \boldsymbol{\beta}_\perp^2(\zeta) - \frac{1}{(z_0-z')}\left(\int_{z'}^{z_0} d\zeta \boldsymbol{\beta}_\perp(\zeta)\right)^2. \qquad (5)$$

A computer code has been developed in order to integrate Eq. (1) at different positions of the test particle within the bunch and at different locations of the bunch within the magnet, obtaining, therefore, the instantaneous rate of energy change of any particle within the bunch for different values of γ.

Consider, as in [4], a rigid, 1D bunch with Gaussian particle density distribution $\lambda(s)$ (s being the coordinate inside the moving bunch) entering a hard edge bending magnet after coming from an infinitely long straight section. The bunch standard deviation will be indicated by $\sigma = 50$ μm, and the total charge will be $q = 1$ nC. In the actual simulation the Gaussian beam will be truncated at $\pm 10 s/\sigma$, where the distribution is understood to be centered at $s/\sigma = 0$, that is,

$$\lambda(s) = \lambda_0 e^{-s^2/2\sigma^2}. \qquad (6)$$

The magnet has hard edges and a curvature radius $R = 1.5$ m.

The rates of energy change of an electron as a function of its position along the Gaussian bunch and at several positions of the bunch after the beginning of the magnet have been

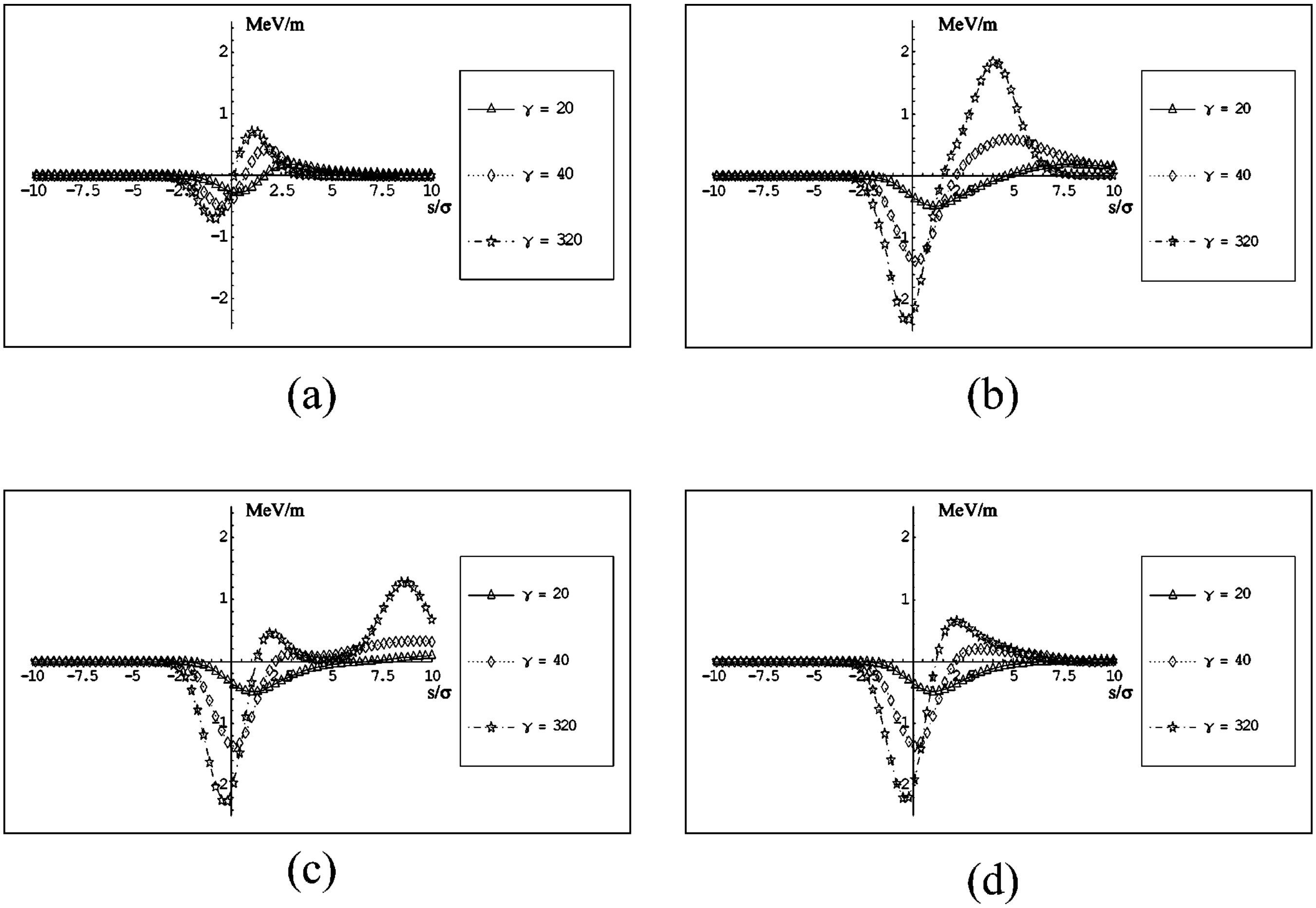

FIG. 2. Rate of energy change of an electron in MeV/m as a function of its position along the Gaussian bunch s/σ entering a hard edge magnet, as calculated using our approach. Every picture shows results for different values of γ. Parameters are $R=1.5$ m, $\sigma=50$ μm, $q=1$ nC. (a) 5 cm after the entrance; (b) 14 cm after the entrance; (c) 18 cm after the entrance; and (d) 25 cm after the entrance.

plotted in Fig. 2. These positions cover all the transient phenomenon, in which there are retarded sources in both the straight section and the bending magnet. Simulation results for different values of γ are plotted in every figure. These values have been chosen large enough to keep the system ultrarelativistic but at the same time small enough (in combination with the bunch length and the radius of curvature of the magnet, as will be discussed in detail in Sec. III) so that the usual regime of applicability of the formula in [4], which was referred to in Sec. I, is abandoned.

As one can see by inspection from Fig. 2, the results are strongly dependent on the beam energy. When γ grows enough (again, this statement will be specified quantitatively in Sec. III), the energy-dependent region is abandoned, and curves in Fig. 2 converge to a γ-independent behavior. Note, in particular, that the case in Fig. 2(d) already belongs to the steady state regime: therefore it can be said that the study up to now shows energy dependence in both the transient and the steady state.

III. GENERALIZATION AND COMPARISON

In order to describe, in the same situation presented in Sec. II, the rate of energy change of an electron within a Gaussian bunch entering a hard edge magnet, the following approximate formula is used in [4]:

$$\frac{d\mathcal{E}}{d(ct)} \simeq -\frac{1}{4\pi\epsilon_0}\frac{2e^2N}{3^{1/3}(2\pi)^{1/2}R^{2/3}\sigma^{4/3}} \times\left[\rho^{-1/3}\left(e^{-(\xi-\rho)^2/2}-e^{-(\xi-4\rho)^2/2}\right) + \int_{\xi-\rho}^{\xi}\frac{d\xi'}{(\xi-\xi')^{1/3}}\frac{d}{d\xi'}e^{-\xi'^2/2}\right], \tag{7}$$

where $\xi=s/\sigma$, $\rho=R\phi^3/24\sigma$, and ϕ is an angle fixing the position of the bunch inside the magnet (see Fig. 3).

It is worth underlining, once again, the importance of Eq. (7), which (in its generalized form for any bunch density distribution function) has been used as a basis for several CSR analysis and computations (see [12–17]).

Equation (7) is completely independent of γ, as one can easily realize by inspection. A comparison between the results that one can obtain by direct integration of Eq. (7) (these results are well known, and presented in [4]) and the results obtained in this paper for the case $\gamma=320$ (reproduced also in Fig. 2) is plotted in Fig. 4.

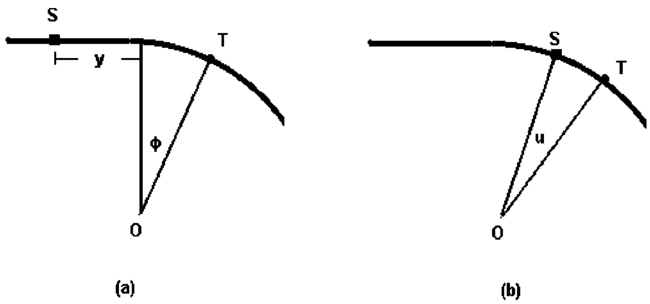

FIG. 3. Geometry for the present position of a test particle T and the retarded position of a source particle S. (a) S is on the straight line. (b) S is in the bend.

It was pointed out in Sec. II that the curves in Fig. 2 saturate, for high values of the Lorentz factor, to a γ-independent behavior: Fig. 4 shows that they reproduce asymptotically, i.e., for large values of γ, the behavior predicted by Eq. (7). The reason why the agreement is only asymptotic lies in the approximations used to derive Eq. (7).

Moreover, it is important to note that, as Fig. 2 and Fig. 4 directly show, an acritical use of the energy-independent Eq. (7) may lead to an overestimation of all CSR longitudinal effects in the energy-dependent regime.

In the following, the hypothesis upon which Eq. (7), taken from [4], was built will be briefly reviewed. By considering the transient phenomenon *in toto*, that is, up to saturation to the steady state regime (when all the retarded sources are in the bend), it has been implicitly assumed that the bending magnet is long enough to allow the entire bunch to enter such a regime. In addition to this assumption, Eq. (7) was derived, in [4], when the following condition is met:

$$\frac{R}{\gamma^3}\frac{d\lambda(s)}{ds} \ll \lambda(s). \tag{8}$$

This basically means that the bunch length is much larger than R/γ^3. For a Gaussian beam with characteristic length σ, Eq. (8) can be written as

$$\gamma \gg \left(\frac{Rs}{\sigma^2}\right)^{1/3}. \tag{9}$$

Taking $s \simeq \sigma$ this means $\gamma \gg 30$.

Moreover, always in [4], the Gaussian beam is treated as a superposition of bunches with rectangular density distribution satisfying the following condition:

$$\hat{\phi}_b \gg 1, \tag{10}$$

where $\hat{\phi}_b$ is defined by the retardation condition

$$\frac{\hat{\phi}_b}{2} + \frac{\hat{\phi}_{b^3}}{24} = \frac{\gamma^3}{R} l_b, \tag{11}$$

where l_b is the bunch length. When Eq. (10) is satisfied, Eq. (11) reads

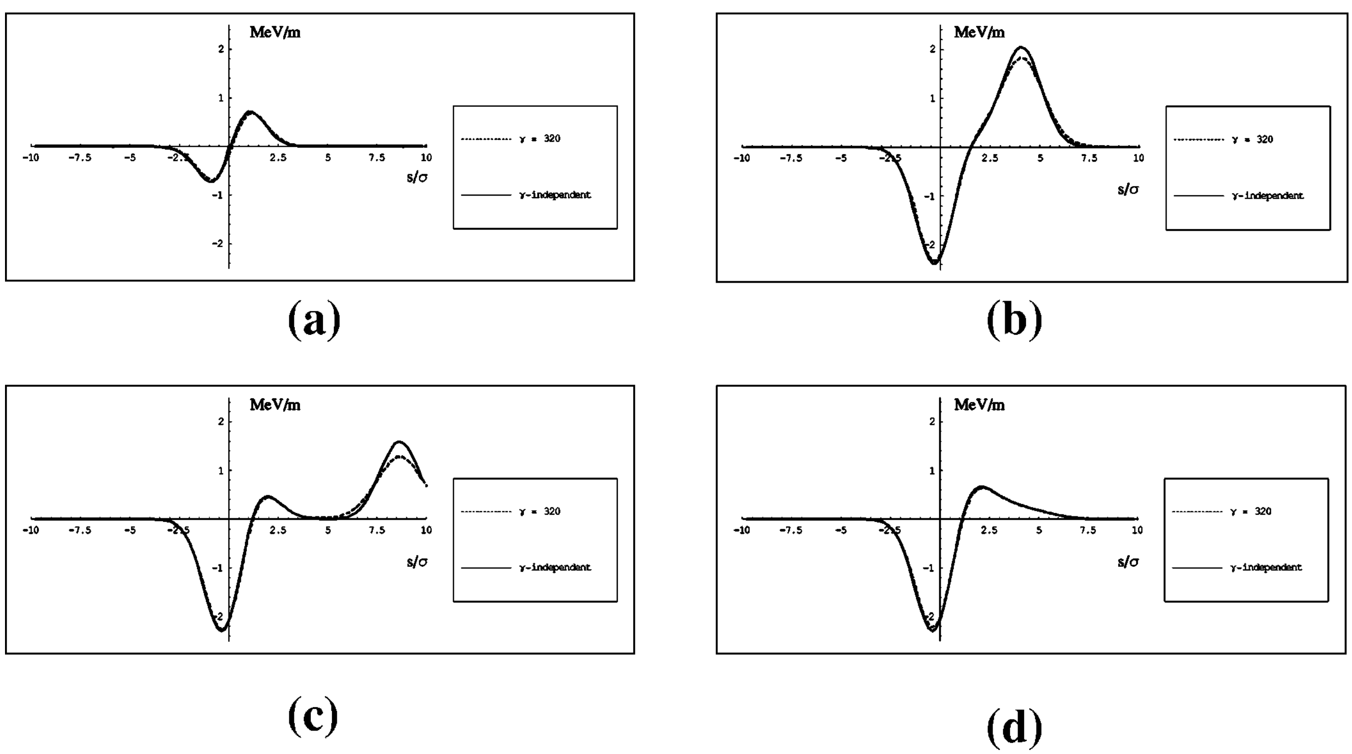

FIG. 4. Comparison between analytical results from literature [4] and our results shown in Fig. 2 for $\gamma = 320$. Rate of energy change of an electron in MeV/m as a function of its position along the Gaussian bunch s/σ entering a hard edge magnet. Parameters are $R = 1.5$ m, $\sigma = 50$ μm, and $q = 1$ nC. The plots are γ independent. (a) 5 cm after the entrance. (b) 14 cm after the entrance. (c) 18 cm after the entrance. (d) 25 cm after the entrance.

$$\left(\frac{24\gamma^3 l_b}{R}\right)^{1/3} \gg 1 \tag{12}$$

or

$$\gamma \gg \left(\frac{R}{24 l_b}\right)^{1/3}. \tag{13}$$

Taking $l_b \simeq \sigma$, this means $\gamma \gg 10$.

From the previous discussion it follows, as a conclusion, that Eq. (8) and Eq. (10) set a lower limit [expressed explicitly by Eq. (9) and Eq. (13)] to the values of γ above which Eq. (7) is valid. Note that this limit is in agreement with the fact that simulation results approach the analytical expectation in [4] for high values of γ, as was already discussed above in this section.

As one more comment, remember that recent research (see [13–15]) has shown that there is reason for concern about beam instabilities induced by CSR in storage rings as well as in bunch compressors. In fact, CSR has been shown to amplify small sinusoidal perturbations of the bunch distribution during the evolution of the beam in magnetic systems: in all these cases, one should make sure that the perturbation wavelength (in combination with the beam energy and the geometry of the magnetic system) satisfies Eq. (8) and Eq. (10) before actually using an energy-independent wake to describe CSR effects.

An extension of Eq. (7) will now be sought that is valid regardless of whether such conditions are met, with the intent of comparing it with the results in Fig. 2. In order to do this consider first Fig. 3, and the equations, also derived in [4], for the rate of energy change of a test particle T at some point in the bend due to the interaction with source particles S whose retarded positions are in the straight line before the bend:

$$\frac{d\mathcal{E}}{d(ct)} = \frac{2e^2\gamma\lambda_0}{4\pi\epsilon_0 R}\int_0^\infty d\hat{y}\left\{\frac{(\hat{\phi}+\hat{y})^2+\hat{\phi}^3(3\hat{\phi}/4+\hat{y})}{[(\hat{\phi}+\hat{y})^2+\hat{\phi}^4/4]^2} - \frac{(\hat{\phi}+\hat{y})^2+\hat{\phi}^4/4}{[(\hat{\phi}+\hat{y})^2+(\hat{\phi}^3/12)(\hat{\phi}/4+\hat{y})]^2}\right\}, \tag{14}$$

or inside the arc:

$$\frac{d\mathcal{E}}{d(ct)} = \frac{2e^2\gamma\lambda_0}{4\pi\epsilon_0 R}\int_0^{\hat{\phi}} d\hat{u}\left(1+\frac{\hat{u}^2}{4}\right)\left\{\frac{\hat{u}^2/4-1}{2(1+\hat{u}^2/4)^3} + \frac{1}{\hat{u}^2}\left[\frac{1+3\hat{u}^2/4}{(1+\hat{u}^2/4)^3} - \frac{1}{(1+\hat{u}^2/12)^2}\right]\right\}. \tag{15}$$

Equations (14) and (15) are valid in the case of an infinitely long electron bunch with constant particle density distribution λ_0. Here, as in [4], $\hat{\phi}=\gamma\phi$, $\hat{y}=y\gamma/R$, and $\hat{u}=\gamma u$ are the meanings of y, ϕ, and u explained in Fig. 3. Moreover, always in [4], we find the following relations, that hold, respectively, when S is in the straight line:

$$(\hat{s}-\hat{s}') = \frac{\hat{\phi}+\hat{y}}{2} + \frac{\hat{\phi}^3}{24}\,\frac{\hat{\phi}+4\hat{y}}{\hat{\phi}+\hat{y}}, \tag{16}$$

or in the bend:

$$(\hat{s}-\hat{s}') = \frac{\hat{u}}{2} + \frac{\hat{u}^3}{24}, \tag{17}$$

where $(\hat{s}-\hat{s}')=(s-s')\gamma^3/R$, and $(s-s')$ is the curvilinear distance between the test particle and a source *at the same time*.

By means of Eqs. (14)–(17), the following extension of Eq. (7) can be written down for the total rate of energy change of an electron entering a bending magnet as a function of its position along the Gaussian bunch:

$$\frac{d\mathcal{E}}{d(ct)} = \frac{2e^2\gamma\lambda_0}{4\pi\epsilon_0 R}\left\{\int_0^\infty d\hat{y}\left[\frac{(\hat{\phi}+\hat{y})^2+\hat{\phi}^3(3\hat{\phi}/4+\hat{y})}{[(\hat{\phi}+\hat{y})^2+\hat{\phi}^4/4]^2} - \frac{(\hat{\phi}+\hat{y})^2+\hat{\phi}^4/4}{[(\hat{\phi}+\hat{y})^2+(\hat{\phi}^3/12)(\hat{\phi}/4+\hat{y})]^2}\right]\right.$$
$$\times e^{-\frac{\{s-(R/\gamma^3)(\hat{\phi}+\hat{y})/2+(\hat{\phi}^3/24)(\hat{\phi}+4\hat{y})/(\hat{\phi}+\hat{y})\}^2}{2\sigma^2}} + \int_0^{\hat{\phi}} d\hat{u}\left(1+\frac{\hat{u}^2}{4}\right)\left[\frac{\hat{u}^2/4-1}{2(1+\hat{u}^2/4)^3}\right.$$
$$\left.\left. + \frac{1}{\hat{u}^2}\left(\frac{1+3\hat{u}^2/4}{(1+\hat{u}^2/4)^3} - \frac{1}{(1+\hat{u}^2/12)^2}\right) e^{-\frac{\{s-(R/\gamma^3)(\hat{u}/2+\hat{u}^3/24)\}^2}{2\sigma^2}}\right]\right\}. \tag{18}$$

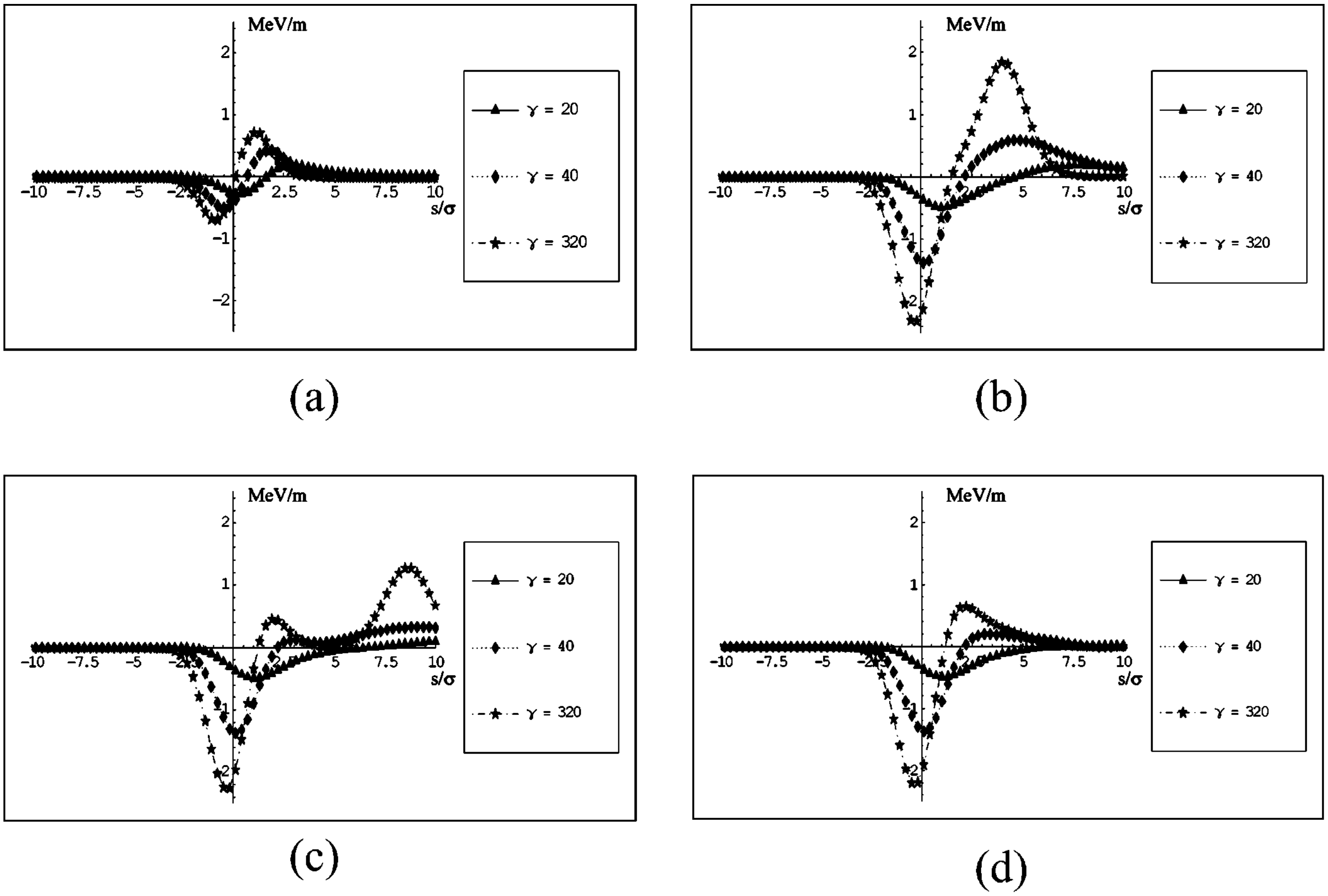

FIG. 5. Rate of energy change of an electron in MeV/m as a function of its position s/σ along the Gaussian bunch entering a hard edge magnet, as calculated using a generalization of Eq. (7). Every picture shows results for different values of γ. Parameters are $R=1.5$ m, $\sigma=50$ μm, $q=1$ nC. (a) 5 cm after the entrance. (b) 14 cm after the entrance. (c) 18 cm after the entrance. (d) 25 cm after the entrance.

Note that the first and second integrals on the right side of Eq. (18) deal, respectively, with retarded sources in the straight line and in the bend. The correct contribution by a source to the rate of energy change of the test particle is given by the integrands of Eq. (14) and Eq. (15) weighted with the particle density distribution [expressed by Eq. (6)], where the latter is evaluated at the retarded position of the source [Eq. (16) and Eq. (17), respectively].

Next, Eq. (18) is integrated, numerically, for the same positions of the beam in the magnet and for the same values of γ that are reported in Fig. 2 and in Fig. 4. Results are shown in Fig. 5.

Note that the main features of Fig. 2 (or Fig. 5) can be explained, in the high-γ case, just by inspecting Eq. (7). For example, the behavior of the maximum in the rate of energy change, present in both simulation results (for $\gamma=320$) and analytical results (see Fig. 4) can be understood by observing that one gets the maximum value at the right-hand side of Eq. (7) when $s=R\phi^3/6$. In fact, for that value of s, one has $\lambda(s-R\phi^3/6)/\lambda_0=e^{-(\xi-4\rho)^2/2}=1$ while the other terms get close to zero. When $R=1.5$ m and the bunch is 14 cm inside the bend the maximum is, therefore, at $s\simeq 4\sigma$; when the bunch is 18 cm inside the bend the maximum has moved to position $s\simeq 8\sigma$, in agreement with the plots. This explains the evolution of the maximum in Fig. 2 for the high-γ case (or Fig. 4, or Fig. 5). By inspecting Fig. 2 and Fig. 5, for the case of lower values of γ, one can see that the peak is still there and still evolving toward the right-hand side of the plots, even if much less pronounced.

In order to get a quantitative evaluation of the agreement between the curves in Fig. 2 and the respective twins in Fig. 5, points have been sampled from every curve in Fig. 2 and in Fig. 5. Then, for every pair of corresponding sampled points (referring to corresponding curves), the differences normalized to the values given by Eq. (7) (shown in Fig. 5) have been taken. Next, the root-mean-square value of these quantities has been considered as a measure of agreement between every pair of twin curves. For every curve agreement within 1% has been found (this small difference is ascribed to computational inaccuracies).

The very good matching between Fig. 2 and Fig. 5 reflects the fact that the general small-angle approximation in [8] was successfully applied to the particular case of a transient between a straight line and a bend: in other words, as expected, Eq. (1) reduces to Eq. (18) when that particular trajectory (straight line followed by a bend) is selected. Of course, both reduce to Eq. (7) as soon as the conditions expressed in Eq. (8) and Eq. (10) are satisfied, as they must.

IV. CONCLUSIONS AND SPECULATIONS

CSR longitudinal transient effects from a straight to a circular path have often been studied under approximations Eq. (8) and Eq. (10), in which they are energy independent. With the help of the method described in [8] the case in which these approximations are no longer valid was addressed instead. In this situation, the results (see Fig. 2) show a strong dependence of the CSR longitudinal force on the Lorentz factor (both in the steady state and in the transient case) and an asymptotic agreement, for large values of γ, with Eq. (7) (see Fig. 4). A conclusion is that, in this energy-dependent regime, the use of the energy-independent Eq. (7) leads to an overall overestimation of all CSR longitudinal effects.

The study proposed in the present paper is of practical interest for low-energy injector design as well as in the framework of CSR-related instabilities: in the first case one considers ultrarelativistic beams whose value of γ (in combination with the bunch length and the choice of the trajectory) may be low enough not to fulfill Eq. (8) and Eq. (10). Similarly, in the second case, when short-wavelength perturbations of the bunch distribution do not meet the requirements in Eq. (8) and Eq. (10), one should use γ-dependent wake fields in order to model CSR interactions. Further investigations may address these practical situations in a more quantitative way.

The results were compared with an extension of the γ-independent formula in [4] for the rate of energy change of an electron [Eq. (7)], and very good agreement between the two outcomes was demonstrated. This reflects the fact that the general approach in [8] perfectly succeeded in dealing with the particular case of a transient. In other words, Eq. (1), valid for any trajectory (under the constraint of the paraxial approach explained in [8]), reduces to Eq. (18) when the correct trajectory (straight line followed by a bend) is selected.

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